B.sc(H) part 2 paper 3
Toppic:Concept of sub group & cycle group with examples
Subject:Mathematics
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Subgroup

Defination : *Let G be a group and H a subset of G. Then H is called to be a subgroup of G if H is a group under the group operation of G.*

Ex. 1 The set of integers I is an additive group. The set E of even integers is a subset of I and is a group under addition.

Hence E is a subgroup of I.

That is, *the set of even integers is a subgroup of the additive group of integers.*

Ex.2. The set Q of rational numbers is a group under addition. The set I of integers is a subset of Q and is a group under addition.

Hence I is a subgroup of Q.

That is, *the set of integers is a subgroup of the additive group of rational numbers.*

Ex.3. The set R of real numbers is a group under addition. The set Q of rational numbers is a subset of R and is a group under addition.

Hence Q is a subgroup of R.

That is, *the set of rational numbers is a subgroup of the additive group of real numbers.*

Ex.4. The set C of complex numbers is a group under addition. The set R of real numbers is a subset of C and is a group under addition. Hence R is a subgroup of C.

That is, *the set of real numbers is a subgroup of the additive group of complex numbers.*

Ex.5. The set R^* of non-zero real numbers is a group under multiplication.

The set Q^* of non-zero rational numbers is a subset of R^* and is a group under multiplication.

Hence Q^* is a subgroup of R^*.

That is, *the set of non zero rational numbers is a subgroup of the multiplicative group of non-zero real numbers.*

Ex.6. The set C^* of non-zero complex numbers is a group under multiplication. The set R^* of non-zero real numbers is a subset of C^* and is a group under multiplication.

Hence R^* is a subgroup of C^*.

That is, *the set of non-zero real numbers is a subgroup of the multiplicative group of non-zero complex numbers.*

Ex.7. The set I of integers is a group under addition. The set P of positive integers is a subset of I but is not a group under addition since no element of P has an inverse in P.

Hence P is not a subgroup of I.

DefinationCyclic group. *A group G is said to be cyclic if there exists $a \in G$ such that the subgroup generated by a is the group itself.*

Thus we may define a cyclic group in this way also.

If a group G contains an element a such that every element of G is of the form a^k for some integer k, we say that G is a cyclic group and that G is generated by a or that a is a generator of G.

Taking additive composition, each element of the cyclic group is some positive or negative multiples of the generator i.e. of the form na where a is the generator.

The fact that G is a cyclic group generated by a is denoted by the symbol $G = \langle a \rangle$.

Ex.1. Prove that the set G consisting of four fourth roots of unity, i.e. $G = \{1, -1, i, -i\}$ is a cyclic group.

Soln. We know that G is a group and now it can be shown that G forms a cyclic group with generators i and $-i$ since.

$i = i$	$-i = -i$
$(i)^2 = -1$	$(-i)^2 = -1$
$(i)^3 = -i$	$(-i)^3 = i$
$(i)^4 = 1$	$(-i)^4 = 1$

It can be readily seen that 1 or −1 cannot be used as generators for G.

Ex.2. Prove that $(Z, +)$ is an infinite cyclic group.

Soln. Here $1 \in Z$ such that $\{n1 : n \in Z\} = \{n : n \in Z\} = Z$.

Hence Z is a cyclic group generated by 1.

Similarly $-1 \in Z$ such that $\{n(-1) : n \in Z\} = \{-n : n \in Z\} = Z$.

Hence Z is a cyclic group generated by −1.

Thus Z is a cyclic group whose generators are +1 and −1.